On the derivation of viscoelastic models for rod-like suspensions

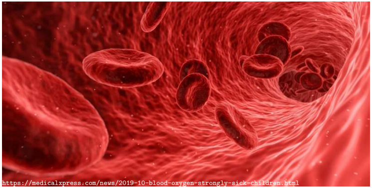
Richard Höfer

Institut de Mathématiques de Jussieu – Paris Rive Gauche
Université Paris Cité

Conférence Singflows
Institut de Mathématiques de Bordeaux, April 13, 2022

Joint work with Marta Leocata (LUISS, Rome)
and Amina Mecherbet (Université Paris Cité)

Suspensions



https://medicalxpress.com/news/2019-10-blood-oxygen-strongly-sick-children.html

https://commons.wikimedia.org/wiki/File:Eyjafjallajokull-April-17.JPG

https://commons.wikimedia.org/wiki/File:Pilt_1.jpg

https://stockhead.com.au/health/avita-wins -us-approval-to-sell-its-spray-on-skin-shares-jump-28pc/

https://www.pinterest.com.mx/pin/843862048921248596/

Doi model for viscoelasticity of dilute rigid suspensions

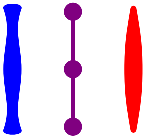
Model for inertialess rod-like Brownian particles with density $f(t,x,\xi)$, $\xi \in S^2$ particle orientation (see e.g. [Doi–Edwards '86]):

$$\partial_t f + u \cdot \nabla_x f + \mathrm{div}_\xi (P_{\xi^\perp}(\xi \cdot \nabla u) f) = D_t \Delta_x f + D_r \Delta_\xi f,$$

$$-\mathrm{div}(\mu_{eff}[f] Du) + \nabla p = h + \mathrm{div}\, \sigma_e, \quad \mathrm{div}\, u = 0,$$

$$\text{Elastic stress: } \sigma_e = \lambda_1 \int_{S^2} (3\xi \otimes \xi - \mathrm{Id}) f \, \mathrm{d}\xi,$$

$$\text{Effective viscosity: } \mu_{eff}[f] = 2 + \lambda_2 \int_{S^2} \xi \otimes \xi \otimes \xi \otimes \xi f \, \mathrm{d}\xi$$



Study of the macroscopic system (well-posedness, numerical simulation, ...) [Constantin '05], [Helzel–Otto–Tzavaras '06,'08,'16,'17], [Masmoudi–Lions '07], [Degond–Liu '09], [Bae–Trivisa '12,'13], [La '18].

Doi model for viscoelasticity of dilute rigid suspensions

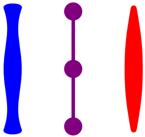
Model for inertialess rod-like Brownian particles with density $f(t, x, \xi)$, $\xi \in S^2$ particle orientation (see e.g. [Doi–Edwards '86]):

$$\partial_t f + u \cdot \nabla_x f + \operatorname{div}_\xi (P_{\xi^\perp}(\xi \cdot \nabla u) f) = D_t \Delta_x f + D_r \Delta_\xi f,$$

$$-\operatorname{div}(\mu_{eff}[f] Du) + \nabla p = h + \operatorname{div} \sigma_e, \quad \operatorname{div} u = 0,$$

Elastic stress: $\sigma_e = \lambda_1 \int_{S^2} (3\xi \otimes \xi - \mathrm{Id}) f \, \mathrm{d}\xi,$

Effective viscosity: $\mu_{eff}[f] = 2 + \lambda_2 \int_{S^2} \xi \otimes \xi \otimes \xi \otimes \xi f \, \mathrm{d}\xi$

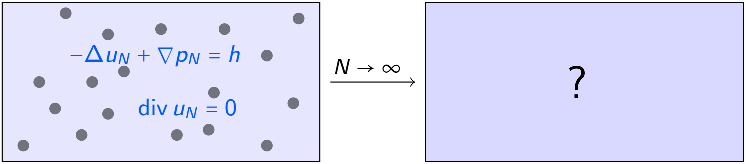


Study of the macroscopic system (well-posedness, numerical simulation, ...) [Constantin '05], [Helzel–Otto–Tzavaras '06,'08,'16,'17], [Masmoudi–Lions '07], [Degond–Liu '09], [Bae–Trivisa '12,'13], [La '18].

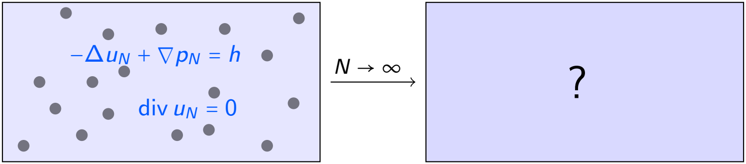
Related models: liquid crystals, elastic polymers, active particles.

Goal: Rigorous derivation of the Doi-model – the elastic stress – from a microscopic fluid-particle-system.

Einstein's effective viscosity formula



Einstein's effective viscosity formula

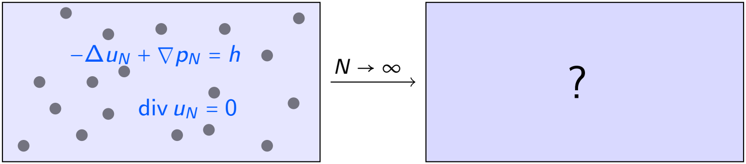


Einstein 1906: For inertialess spherical particles the suspension can be described by an effective viscosity

$$\mu_{eff} = 1 + \frac{5}{2}\phi + o(\phi),$$

where ϕ is the particle volume fraction.

Einstein's effective viscosity formula



Einstein 1906: For inertialess spherical particles the suspension can be described by an effective viscosity

$$\mu_{eff} = 1 + \frac{5}{2}\phi + o(\phi),$$

where ϕ is the particle volume fraction.
More precisely, under straining boundary conditions

$$u_N = Ex \quad \text{on } \partial\Omega, \qquad E \in \mathrm{Sym}_0(3)$$

with no external forces, the energy dissipation satisfies

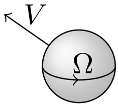
$$\int_\Omega |Du_N|^2 = |\Omega||E|^2 \left(1 + \frac{5}{2}\phi + o(\phi)\right)$$

Resistance and mobility of a single particle

$$-\Delta u + \nabla p = 0, \quad \operatorname{div} u = 0 \quad \text{in } \mathbb{R}^3 \smallsetminus \mathcal{B},$$

$$u(x) = V + \Omega \times (x - X) \quad \text{in } \mathcal{B}.$$

$$\lim_{|x| \to \infty} u(x) - (u_\infty + \omega_\infty \wedge (x - X) + E_\infty (x - X)) = 0.$$

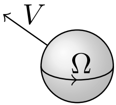


$$F = -\int_{\partial \mathcal{B}} \sigma[u, p]\nu, \qquad T = -\int_{\partial \mathcal{B}} \sigma[u, p]\nu \wedge (x - X),$$

$$S = -\mathrm{sym}_0 \left(\int_{\partial \mathcal{B}} \sigma[u, p]\nu \otimes (x - X) \right),$$

Resistance and mobility of a single particle

$$-\Delta u + \nabla p = 0, \quad \operatorname{div} u = 0 \quad \text{in } \mathbb{R}^3 \setminus \mathcal{B},$$
$$u(x) = V + \Omega \times (x - X) \quad \text{in } \mathcal{B}.$$
$$\lim_{|x|\to\infty} u(x) - (u_\infty + \omega_\infty \wedge (x - X) + E_\infty(x - X)) = 0.$$



$$F = -\int_{\partial\mathcal{B}} \sigma[u,p]\nu, \qquad T = -\int_{\partial\mathcal{B}} \sigma[u,p]\nu \wedge (x - X),$$
$$S = -\mathrm{sym}_0\left(\int_{\partial\mathcal{B}} \sigma[u,p]\nu \otimes (x - X)\right),$$

- Resistance problem:
  Given $V, \Omega, u_\infty, \omega_\infty \in \mathbb{R}^3$ and $E_\infty \in Sym_0(3)$,
  find F, T, S

$$\begin{pmatrix} F \\ T \\ S \end{pmatrix} = \mathcal{R}_\mathcal{B} \begin{pmatrix} V - u_\infty \\ \Omega - \omega_\infty \\ -E_\infty \end{pmatrix}.$$

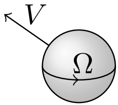
- Mobility problem:
  Given $F, T, u_\infty, \omega_\infty \in \mathbb{R}^3$ and $E_\infty \in Sym_0(3)$,
  find V, Ω, S

$$\begin{pmatrix} V - u_\infty \\ \Omega - \omega_\infty \\ S \end{pmatrix} = \mathcal{M}_\mathcal{B} \begin{pmatrix} F \\ T \\ -E_\infty \end{pmatrix}.$$

Resistance and mobility of a single particle

$$-\Delta u + \nabla p = 0, \quad \operatorname{div} u = 0 \quad \text{in } \mathbb{R}^3 \setminus \mathcal{B},$$
$$u(x) = V + \Omega \times (x - X) \quad \text{in } \mathcal{B}.$$

$$\lim_{|x|\to\infty} u(x) - (u_\infty + \omega_\infty \wedge (x - X) + E_\infty(x - X)) = 0.$$



$$F = -\int_{\partial\mathcal{B}} \sigma[u,p]\nu, \qquad T = -\int_{\partial\mathcal{B}} \sigma[u,p]\nu \wedge (x - X),$$
$$S = -\mathrm{sym}_0\left(\int_{\partial\mathcal{B}} \sigma[u,p]\nu \otimes (x - X)\right),$$

- Resistance problem:
  Given $V, \Omega, u_\infty, \omega_\infty \in \mathbb{R}^3$ and $E_\infty \in Sym_0(3)$,
  find F, T, S

$$\begin{pmatrix} F \\ T \\ S \end{pmatrix} = \mathcal{R}_{\mathcal{B}} \begin{pmatrix} V - u_\infty \\ \Omega - \omega_\infty \\ -E_\infty \end{pmatrix}.$$

- Mobility problem:
  Given $F, T, u_\infty, \omega_\infty \in \mathbb{R}^3$ and $E_\infty \in Sym_0(3)$,
  find V, Ω, S

$$\begin{pmatrix} V - u_\infty \\ \Omega - \omega_\infty \\ S \end{pmatrix} = \mathcal{M}_{\mathcal{B}} \begin{pmatrix} F \\ T \\ -E_\infty \end{pmatrix}.$$

Sphere of radius R: $\mathcal{M}_{B_R}$ is diagonal.

In particular: $F = T = 0 \quad \Rightarrow \quad V = u_\infty, \; \Omega = \omega_\infty, \; S = -\frac{5}{2}\frac{4\pi}{3}R^3 E_\infty$

Formal homogenization through superposition

- N identical spherical particles $B_i = B_R(X_i)$,
- $\frac{1}{N}\sum_i \delta_{X_i} \to \rho$, $\phi = \frac{4\pi}{3} N R^3 \ll 1$

$$
\begin{aligned}
-\Delta u_N + \nabla p_N = h, \quad \operatorname{div} u_N &= 0 \qquad \text{in } \mathbb{R}^3 \setminus \cup_i B_i, \\
D u_N &= 0 \qquad \text{in } \cup_i B_i, \\
\int_{\partial B_i} \sigma[u_N, p_N]\nu = 0, &\qquad \int_{\partial B_i} \sigma[u_N, p_N]\nu \wedge (x - X_i) = 0,
\end{aligned}
$$

Formal homogenization through superposition

- N identical spherical particles $B_i = B_R(X_i)$,
- $\frac{1}{N} \sum_i \delta_{X_i} \to \rho$, $\phi = \frac{4\pi}{3} N R^3 \ll 1$

$$\begin{aligned} -\Delta u_N + \nabla p_N = h, \quad \operatorname{div} u_N &= 0 && \text{in } \mathbb{R}^3 \setminus \cup_i B_i, \\ D u_N &= 0 && \text{in } \cup_i B_i, \\ \int_{\partial B_i} \sigma[u_N, p_N]\nu &= 0, && \int_{\partial B_i} \sigma[u_N, p_N]\nu \wedge (x - X_i) = 0, \end{aligned}$$

Energy estimate: $u_N \rightharpoonup u_{eff}$ in $\dot{H}^1(\mathbb{R}^3)$.

Formal homogenization through superposition

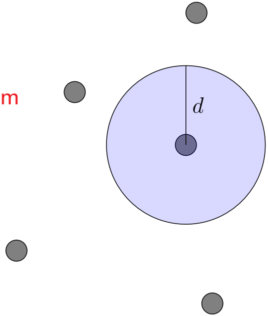
- N identical spherical particles $B_i = B_R(X_i)$,
- $\frac{1}{N}\sum_i \delta_{X_i} \to \rho$, $\phi = \frac{4\pi}{3} N R^3 \ll 1$

$$
\begin{aligned}
-\Delta u_N + \nabla p_N = h, \quad \operatorname{div} u_N &= 0 \qquad \text{in } \mathbb{R}^3 \setminus \cup_i B_i, \\
D u_N &= 0 \qquad \text{in } \cup_i B_i, \\
\int_{\partial B_i} \sigma[u_N, p_N]\nu &= 0, \qquad \int_{\partial B_i} \sigma[u_N, p_N]\nu \wedge (x - X_i) = 0,
\end{aligned}
$$

Energy estimate: $u_N \rightharpoonup u_{eff}$ in $\dot{H}^1(\mathbb{R}^3)$.

In $B_d(X_i)$, $R \ll d \ll N^{-1/3}$: approximate mobility problem with $F = 0$, $T = 0$, $E_\infty = D u_{eff}(X_i)$.
Thus, approximate

$$
-\Delta u_N + \nabla p_N \approx h - \sum_i \frac{5}{2}\frac{4\pi}{3} R^3 D u_{eff}(X_i) \nabla \delta_{X_i}
$$



Formal homogenization through superposition

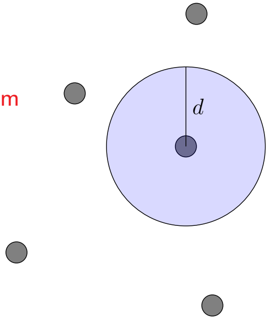
- N identical spherical particles $B_i = B_R(X_i)$,
- $\frac{1}{N}\sum_i \delta_{X_i} \to \rho$, $\phi = \frac{4\pi}{3} N R^3 \ll 1$

$$\begin{aligned} -\Delta u_N + \nabla p_N = h, \quad \operatorname{div} u_N &= 0 \qquad \text{in } \mathbb{R}^3 \setminus \cup_i B_i, \\ D u_N &= 0 \qquad \text{in } \cup_i B_i, \\ \int_{\partial B_i} \sigma[u_N, p_N]\nu = 0, &\qquad \int_{\partial B_i} \sigma[u_N, p_N]\nu \wedge (x - X_i) = 0, \end{aligned}$$

Energy estimate: $u_N \rightharpoonup u_{eff}$ in $\dot{H}^1(\mathbb{R}^3)$.

In $B_d(X_i)$, $R \ll d \ll N^{-1/3}$: approximate mobility problem with $F = 0$, $T = 0$, $E_\infty = D u_{eff}(X_i)$.
Thus, approximate

$$-\Delta u_N + \nabla p_N \approx h - \sum_i \frac{5}{2}\frac{4\pi}{3} R^3 D u_{eff}(X_i) \nabla \delta_{X_i}$$



Taking limits on both sides

$$-\Delta u_{eff} + \nabla p \approx h - \frac{5}{2}\phi \operatorname{div}(2\rho D u_{eff})$$

Rigorous mathematical results

- Haines–Mazzucato '11: Energy dissipation formula for periodic configurations

Rigorous mathematical results

- Haines–Mazzucato '11: Energy dissipation formula for periodic configurations
- Schubert–Niethammer '19, Gérard-Varet '20, Gérard-Varet–H. '21: Homogenization results.

Rigorous mathematical results

- Haines–Mazzucato '11: Energy dissipation formula for periodic configurations
- Schubert–Niethammer '19, Gérard-Varet '20, Gérard-Varet–H. '21: Homogenization results.
- Hillairet–Wu '20: Non spherical particles: μ_{eff} is tensor $\mathrm{Sym}_0(3) \to \mathrm{Sym}_0(3)$ depending on the particle shapes.

Rigorous mathematical results

- Haines–Mazzucato '11: Energy dissipation formula for periodic configurations
- Schubert–Niethammer '19, Gérard-Varet '20, Gérard-Varet–H. '21: Homogenization results.
- Hillairet–Wu '20: Non spherical particles: μ_{eff} is tensor $\mathrm{Sym}_0(3) \to \mathrm{Sym}_0(3)$ depending on the particle shapes.
- Kozlov '93, Duerinckx–Gloria '20, Duerinckx '20, Gerard-Varet–Girodroux-Lavigne '21 Existence of an effective viscosity tensor.

Rigorous mathematical results

- Haines–Mazzucato '11: Energy dissipation formula for periodic configurations
- Schubert–Niethammer '19, Gérard-Varet '20, Gérard-Varet–H. '21: Homogenization results.
- Hillairet–Wu '20: Non spherical particles: μ_{eff} is tensor $\mathrm{Sym}_0(3) \to \mathrm{Sym}_0(3)$ depending on the particle shapes.
- Kozlov '93, Duerinckx–Gloria '20, Duerinckx '20, Gerard-Varet–Girodroux-Lavigne '21 Existence of an effective viscosity tensor.
- Gérard-Varet–Hillairet '20, Duerinckx–Gloria '20, Gérard-Varet–Mecherbet '20, Gérard-Varet '20: Second order and higher order expansions:

$$\mu_{eff} = \mathrm{Id} + \frac{5}{2}\phi \mathrm{Id} + \mu_2 \phi^2 + \dots$$

Rigorous mathematical results

- Haines–Mazzucato '11: Energy dissipation formula for periodic configurations
- Schubert–Niethammer '19, Gérard-Varet '20, Gérard-Varet–H. '21: Homogenization results.
- Hillairet–Wu '20: Non spherical particles: μ_{eff} is tensor $\mathrm{Sym}_0(3) \to \mathrm{Sym}_0(3)$ depending on the particle shapes.
- Kozlov '93, Duerinckx–Gloria '20, Duerinckx '20, Gerard-Varet–Girodroux-Lavigne '21 Existence of an effective viscosity tensor.

- Gérard-Varet–Hillairet '20, Duerinckx–Gloria '20, Gérard-Varet–Mecherbet '20, Gérard-Varet '20: Second order and higher order expansions:

$$\mu_{eff} = \mathrm{Id} + \frac{5}{2}\phi \mathrm{Id} + \mu_2 \phi^2 + \dots$$

- H'.–Schubert '21: Time evolution of (buoyant, sedimenting) particles:

$$\partial_t \rho + (u + \gamma^{-1} g) \nabla \rho = 0$$

$$-\operatorname{div}\left(2\left(1 + \frac{5}{2}\phi\rho\right) Du\right) + \nabla p = \rho g.$$

Back to viscoelasticity

Model for inertialess rod-like Brownian particles with density $f(t,x,\xi)$, $\xi \in S^2$ particle orientation (see e.g. [Doi–Edwards '86]):

$$\partial_t f + u \cdot \nabla_x f + \operatorname{div}_\xi (P_{\xi^\perp}(\xi \cdot \nabla u) f) = D_t \Delta_x f + D_r \Delta_\xi f,$$

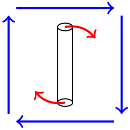
$$-\operatorname{div}(\mu_{eff}[f] Du) + \nabla p = h + \operatorname{div} \sigma_e, \quad \operatorname{div} u = 0,$$

$$\text{Elastic stress: } \sigma_e = \lambda_1 \int_{S^2} (3\xi \otimes \xi - \mathrm{Id}) f \, \mathrm{d}\xi,$$

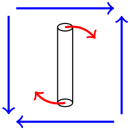
$$\text{Effective viscosity: } \mu_{eff}[f] = 2 + \lambda_2 \int_{S^2} \xi \otimes \xi \otimes \xi \otimes \xi f \, \mathrm{d}\xi$$

Goal: Rigorous derivation of the Doi-model – the elastic stress – from a microscopic fluid-particle-system.

Back to viscoelasticity



Model for inertialess rod-like Brownian particles with density $f(t,x,\xi)$, $\xi \in S^2$ particle orientation (see e.g. [Doi–Edwards '86]):

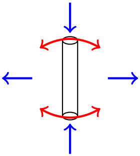


$$\partial_t f + u \cdot \nabla_x f + \mathrm{div}_\xi (P_{\xi^\perp}(\xi \cdot \nabla u) f) = D_t \Delta_x f + D_r \Delta_\xi f,$$

$$-\mathrm{div}(\mu_{eff}[f] Du) + \nabla p = h + \mathrm{div}\, \sigma_e, \quad \mathrm{div}\, u = 0,$$

$$\text{Elastic stress: } \sigma_e = \lambda_1 \int_{S^2} (3\xi \otimes \xi - \mathrm{Id}) f \, \mathrm{d}\xi,$$

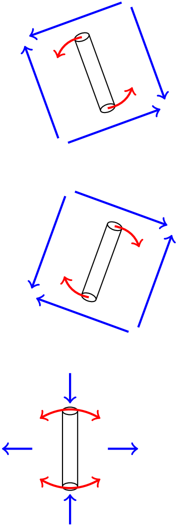
$$\text{Effective viscosity: } \mu_{eff}[f] = 2 + \lambda_2 \int_{S^2} \xi \otimes \xi \otimes \xi \otimes \xi f \, \mathrm{d}\xi$$



Goal: Rigorous derivation of the Doi-model – the elastic stress – from a microscopic fluid-particle-system.

Back to viscoelasticity

Model for inertialess rod-like Brownian particles with density $f(t,x,\xi)$, $\xi \in S^2$ particle orientation (see e.g. [Doi–Edwards '86]):



$$\partial_t f + u\cdot\nabla_x f + \operatorname{div}_\xi(P_{\xi^\perp}(\xi\cdot\nabla u)f) = D_t\Delta_x f + D_r\Delta_\xi f,$$

$$-\operatorname{div}(\mu_{eff}[f]Du) + \nabla p = h + \operatorname{div}\sigma_e, \quad \operatorname{div} u = 0,$$

$$\text{Elastic stress: } \sigma_e = \lambda_1 \int_{S^2} (3\xi\otimes\xi - \mathrm{Id}) f \, \mathrm{d}\xi,$$

$$\text{Effective viscosity: } \mu_{eff}[f] = 2 + \lambda_2 \int_{S^2} \xi\otimes\xi\otimes\xi\otimes\xi f \, \mathrm{d}\xi$$

Goal: Rigorous derivation of the Doi-model – the elastic stress – from a microscopic fluid-particle-system.

A single Brownian particle

Mobility problem

$$-\Delta u + \nabla p = 0, \quad \operatorname{div} u = 0 \quad \text{in } \mathbb{R}^3 \setminus \mathcal{B},$$
$$u = V + \Omega \times (x - X) \quad \text{in } \mathcal{B}.$$
$$\lim_{|x| \to \infty} u(x) = 0.$$

with random forces F^B, T^B given through a 6-dim Brownian motion according to the fluctuation-dissipation-theorem (Stokes-Einstein relation)

$$(F^B, T^B) = \sqrt{2 k_B \Theta \mathcal{R}_{red}} \circ \mathrm{d}B,$$

$$\mathcal{R}_{\mathcal{B}} = \begin{pmatrix} \mathcal{R}_1 & \mathcal{R}_{12} & \mathcal{R}_{13} \\ \mathcal{R}_{12}^T & \mathcal{R}_2 & \mathcal{R}_{23} \\ \mathcal{R}_{13}^T & \mathcal{R}_{23}^T & \mathcal{R}_3 \end{pmatrix}.$$

A single Brownian particle

Mobility problem

$$-\Delta u + \nabla p = 0, \quad \operatorname{div} u = 0 \quad \text{in } \mathbb{R}^3 \setminus \mathcal{B},$$
$$u = V + \Omega \times (x - X) \quad \text{in } \mathcal{B}.$$
$$\lim_{|x|\to\infty} u(x) = 0.$$

with random forces F^B, T^B given through a 6-dim Brownian motion according to the fluctuation-dissipation-theorem (Stokes-Einstein relation)

$$(F^B, T^B) = \sqrt{2k_B\Theta\mathcal{R}_{red}} \circ \mathrm{d}B,$$

For Axisymmetric particles $\mathcal{B}_R = RO(\xi)\mathcal{B}$ with orientation ξ, $O(\xi) \in SO(3)$:

$$\mathcal{R}_{\mathcal{B}_R} = \begin{pmatrix} R\mathcal{R}_1(\xi) & 0 & 0 \\ 0 & R^3\mathcal{R}_2(\xi) & R^3\mathcal{R}_{23}(\xi) \\ 0 & R^3\mathcal{R}_{23}^T(\xi) & R^3\mathcal{R}_3(\xi) \end{pmatrix}.$$

A single Brownian particle

Mobility problem

$$-\Delta u + \nabla p = 0, \quad \operatorname{div} u = 0 \quad \text{in } \mathbb{R}^3 \setminus \mathcal{B},$$
$$u = V + \Omega \times (x - X) \quad \text{in } \mathcal{B}.$$
$$\lim_{|x|\to\infty} u(x) = 0.$$

with random forces F^B, T^B given through a 6-dim Brownian motion according to the fluctuation-dissipation-theorem (Stokes-Einstein relation)

$$(F^B, T^B) = \sqrt{2k_B\Theta\mathcal{R}_{red}} \circ \mathrm{d}B,$$

For Axisymmetric particles $\mathcal{B}_R = RO(\xi)\mathcal{B}$ with orientation ξ, $O(\xi) \in SO(3)$:

$$\mathcal{R}_{\mathcal{B}_R} = \begin{pmatrix} R\mathcal{R}_1(\xi) & 0 & 0 \\ 0 & R^3\mathcal{R}_2(\xi) & R^3\mathcal{R}_{23}(\xi) \\ 0 & R^3\mathcal{R}_{23}^T(\xi) & R^3\mathcal{R}_3(\xi) \end{pmatrix}.$$

Thus, in a fluid at rest,

$$V = R^{-1}\mathcal{R}_1^{-1}(\xi)F^B = R^{-1/2}\sqrt{\mathcal{R}_1^{-1}(\xi)} \circ \mathrm{d}B^t, \qquad \Omega = R^{-3/2}\sqrt{\mathcal{R}_2^{-1}(\xi)} \circ \mathrm{d}B^r$$
$$S = R^3\mathcal{R}_{23}^T(\xi)\Omega = R_{23}^T(\xi)\sqrt{\mathcal{R}_2^{-1}(\xi)} \circ \mathrm{d}B^r$$

Microscopic model of N Brownian particles

- N identical axissymmetric particles $\mathcal{B}_i = O(\xi_i) R\mathcal{B}$ with centers X_i and orientations ξ_i, orientation matrix $O(\xi_i) \in SO(3)$ and $R = R_N \to 0$, $\phi = NR^3 \ll 1$.

$$
\begin{aligned}
-\Delta u_N + \nabla p_N = h, \quad \operatorname{div} u_N = 0 \qquad & \text{in } \mathbb{R}^3 \setminus \cup_i \mathcal{B}_i, \\
u_N(x) = V_i + (x - X_i) \wedge \Omega_i \qquad & \text{in } \cup_i \mathcal{B}_i, \\
\int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu = F_i^B, \qquad & \int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu \wedge (x - X_i) = T_i^B, \\
(F_i^B, T_i^B)_i = \sqrt{\mathcal{R}_N} \circ \mathrm{d}B, \qquad & \\
\dot{X}_i = V_i, \qquad & \dot{\xi}_i = \Omega_i \wedge \xi_i.
\end{aligned}
$$

Microscopic model of N Brownian particles

- N identical axissymmetric particles $\mathcal{B}_i = O(\xi_i)R\mathcal{B}$ with centers X_i and orientations ξ_i, orientation matrix $O(\xi_i) \in SO(3)$ and $R = R_N \to 0$, $\phi = NR^3 \ll 1$.

$$
\begin{aligned}
-\Delta u_N + \nabla p_N = h, \quad \operatorname{div} u_N = 0 \qquad & \text{in } \mathbb{R}^3 \setminus \cup_i \mathcal{B}_i, \\
u_N(x) = V_i + (x - X_i) \wedge \Omega_i \qquad & \text{in } \cup_i \mathcal{B}_i, \\
\int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu = F_i^B, \qquad & \int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu \wedge (x - X_i) = T_i^B, \\
(F_i^B, T_i^B)_i = \sqrt{\mathcal{R}_N} \circ \mathrm{d}B, \qquad & \\
\dot{X}_i = V_i, \qquad & \dot{\xi}_i = \Omega_i \wedge \xi_i.
\end{aligned}
$$

$$
\begin{aligned}
\partial_t f + u \cdot \nabla_x f + \operatorname{div}_\xi (P_{\xi^\perp}(\xi \cdot \nabla u) f) &= D_t \Delta_x f + D_r \Delta_\xi f, \\
-\operatorname{div}(\mu_{eff}[f] Du) + \nabla p = h + \operatorname{div} \sigma_e, &\quad \operatorname{div} u = 0.
\end{aligned}
$$

Microscopic model of N Brownian particles

- N identical axissymmetric particles $\mathcal{B}_i = O(\xi_i) R\mathcal{B}$ with centers X_i and orientations ξ_i, orientation matrix $O(\xi_i) \in SO(3)$ and $R = R_N \to 0$, $\phi = NR^3 \ll 1$.

$$
\begin{aligned}
-\Delta u_N + \nabla p_N = h, \quad \operatorname{div} u_N = 0 \qquad & \text{in } \mathbb{R}^3 \setminus \cup_i \mathcal{B}_i, \\
u_N(x) = V_i + (x - X_i) \wedge \Omega_i \qquad & \text{in } \mathcal{B}_i, \\
\int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu = 0, \qquad & \int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu \wedge (x - X_i) = T_i^B, \\
(F_i^B, T_i^B)_i = \sqrt{\mathcal{R}_N} \circ \mathrm{d}B, \qquad & \\
\dot{X}_i = 0, \qquad & \dot{\xi}_i = \Omega_i \wedge \xi_i.
\end{aligned}
$$

$$
\begin{aligned}
\partial_t f + \cancel{u \cdot \nabla_x f} + \operatorname{div}_\xi (P_{\xi^\perp}(\xi \cdot \nabla u) f) &= \cancel{D_t \Delta_x f} + D_r \Delta_\xi f, \\
-\operatorname{div}(\mu_{eff}[f] Du) + \nabla p &= h + \operatorname{div} \sigma_e, \quad \operatorname{div} u = 0.
\end{aligned}
$$

(Artificial) simplifications:

- Freeze particle translations and neglect Brownian *force*
-
-

Microscopic model of N Brownian particles

- N identical axissymmetric particles $\mathcal{B}_i = O(\xi_i) R \mathcal{B}$ with centers X_i and orientations ξ_i, orientation matrix $O(\xi_i) \in SO(3)$ and $R = R_N \to 0$, $\phi = NR^3 \ll 1$.

$$
\begin{aligned}
-\Delta u_N + \nabla p_N = h, \quad \operatorname{div} u_N = 0 \qquad & \text{in } \mathbb{R}^3 \setminus \cup_i \mathcal{B}_i, \\
u_N(x) = V_i + (x - X_i) \wedge \Omega_i \qquad & \text{in } \mathcal{B}_i, \\
\int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu = 0, \qquad & \int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu \wedge (x - X_i) = T_i^B, \\
T_i^B = \sqrt{R^3 \mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i, \qquad & \\
\dot{X}_i = 0, \qquad & \dot{\xi}_i = \xi_i \wedge \sqrt{R^{-3} \mathcal{R}_2^{-1}(\xi_i)}.
\end{aligned}
$$

$$
\begin{aligned}
\partial_t f + \cancel{u \cdot \nabla_x f} + \cancel{\operatorname{div}_\xi (P_{\xi^\perp}(\xi \cdot \nabla u) f)} = \cancel{D_t \Delta_x f} + D_r \Delta_\xi f, \\
-\operatorname{div}(\mu_{eff}[f] Du) + \nabla p = h + \operatorname{div} \sigma_e, \quad \operatorname{div} u = 0.
\end{aligned}
$$

(Artificial) simplifications:

- Freeze particle translations and neglect Brownian *force*
- No hydrodynamic interaction for particle evolution
-

Microscopic model of N Brownian particles

- N identical axissymmetric particles $\mathcal{B}_i = O(\xi_i) R \mathcal{B}$ with centers X_i and orientations ξ_i, orientation matrix $O(\xi_i) \in SO(3)$ and $R = R_N \to 0$, $\phi = NR^3 \ll 1$.

$$
\begin{aligned}
-\Delta u_N + \nabla p_N = 0, \quad \operatorname{div} u_N = 0 \qquad & \text{in } \mathbb{R}^3 \setminus \cup_i \mathcal{B}_i, \\
u_N(x) = V_i + (x - X_i) \wedge \Omega_i \qquad & \text{in } \mathcal{B}_i, \\
\int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu = 0, \qquad & \int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu \wedge (x - X_i) = T_i^B, \\
T_i^B = \sqrt{R^3 \mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i, \qquad & \\
\dot{X}_i = 0, \qquad & \dot{\xi}_i = \xi_i \wedge \sqrt{R^{-3}\mathcal{R}_2^{-1}(\xi_i)} + P_{\xi_i^\perp} h(t, X_i, \xi_i).
\end{aligned}
$$

$$
\begin{aligned}
\partial_t f + \cancel{u \cdot \nabla_x f} + \cancel{\operatorname{div}_\xi (P_{\xi^\perp}(\xi \cdot \nabla u) f)} = \cancel{D_t \Delta_x f} + D_r \Delta_\xi f - \operatorname{div}_\xi (P_{\xi^\perp} h f), \\
-\operatorname{div}(\mu_{eff}[f] Du) + \nabla p = \cancel{h} + \operatorname{div} \sigma_e, \quad \operatorname{div} u = 0.
\end{aligned}
$$

(Artificial) simplifications:

- Freeze particle translations and neglect Brownian *force*
- No hydrodynamic interaction for particle evolution
- Put source term on the particle evolution only

Microscopic model of N Brownian particles

- N identical axissymmetric particles $\mathcal{B}_i = O(\xi_i) R\mathcal{B}$ with centers X_i and orientations ξ_i, orientation matrix $O(\xi_i) \in SO(3)$ and $R = R_N \to 0$, $\phi = NR^3 \ll 1$.

$$
\begin{aligned}
-\Delta u_N + \nabla p_N = 0, \quad \operatorname{div} u_N &= 0 \qquad \text{in } \mathbb{R}^3 \setminus \cup_i \mathcal{B}_i, \\
D u_N(x) &= 0 \qquad \text{in } \cup_i \mathcal{B}_i, \\
\int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu &= 0, \qquad \int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu \wedge (x - X_i) = R^3 \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i, \\
\dot{X}_i &= 0, \qquad \dot{\xi}_i = \xi_i \wedge \sqrt{\mathcal{R}_2^{-1}(\xi_i)} \circ \mathrm{d}B_i + P_{\xi_i^\perp} h(t, X_i, \xi_i).
\end{aligned}
$$

$$
\begin{aligned}
\partial_t f + \operatorname{div}_\xi((P_{\xi^\perp} hf - \nabla_\xi f) &= 0, \qquad \text{Deborah number } De = 1 \\
-\operatorname{div}(\mu_{eff}[f] Du) + \nabla p = \phi \operatorname{div} \sigma_e, \quad \operatorname{div} u &= 0.
\end{aligned}
$$

(Artificial) simplifications:

- Freeze particle translations and neglect Brownian *force*
- No hydrodynamic interaction for particle evolution
- Put source term on the particle evolution only

Microscopic model of N Brownian particles

- N identical axissymmetric particles $\mathcal{B}_i = O(\xi_i) R \mathcal{B}$ with centers X_i and orientations ξ_i, orientation matrix $O(\xi_i) \in SO(3)$ and $R = R_N \to 0$, $\phi = NR^3 \ll 1$.

$$
\begin{aligned}
-\Delta u_N + \nabla p_N = 0, \quad \operatorname{div} u_N = 0 \qquad & \text{in } \mathbb{R}^3 \setminus \cup_i \mathcal{B}_i, \\
D u_N(x) = 0 \qquad & \text{in } \cup_i \mathcal{B}_i, \\
\int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu = 0, \qquad & \int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu \wedge (x - X_i) = \frac{1}{N} \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i, \\
\dot{X}_i = 0, \qquad & \dot{\xi}_i = \xi_i \wedge \sqrt{\phi^{-1} \mathcal{R}_2^{-1}(\xi_i)} \circ \mathrm{d}B_i + P_{\xi_i^\perp} \phi^{-1} h(t, X_i, \xi_i).
\end{aligned}
$$

$$
\begin{aligned}
\phi \partial_t f + \operatorname{div}_\xi ((P_{\xi^\perp} h f - \nabla_\xi f) = 0, \qquad & \text{Deborah number } De = \phi \\
-\operatorname{div}(\mu_{eff}[f] Du) + \nabla p = \operatorname{div} \sigma_e, \quad \operatorname{div} u = 0. &
\end{aligned}
$$

(Artificial) simplifications:

- Freeze particle translations and neglect Brownian *force*
- No hydrodynamic interaction for particle evolution
- Put source term on the particle evolution only

Statement of the result

- well separated particles: $\min_{i\neq j}|X_i - X_j| \geq cN^{-\frac{1}{3}}$
- small volume fraction: $\phi_N \log N = NR_N^3 \log N \to 0$
- Convergence of initial data: $\mathbb{E}[\mathcal{W}_1(\frac{1}{N}\sum \delta_{X_i,\xi_i^0}, f_0)] \to 0$, $f_0 \in L^2(\mathbb{R}^3 \times \mathbb{S}^2)$

Theorem 1: For $De = 1$, for all $t > 0$, $s > 1/2$

$$\forall \varepsilon > 0 \ \lim_{N\to\infty} \mathbb{P}[\|\phi_N^{-1} u_N - u\|_{H^{-s}((0,t),H^{-s}(\mathbb{R}^3))} + \sup_{\tau\in[0,t]} \mathcal{W}_1(f_N(\tau), f(\tau)) > \varepsilon] = 0,$$

where (f, u) solve

$$\partial_t f + \mathrm{div}_\xi((P_{\xi^\perp} hf - \nabla_\xi f) = 0 \qquad f(0,\cdot) = f_0$$

$$-\Delta u + \nabla p = \gamma_{\mathcal{B}} \int_{\mathbb{S}^2} (3\xi \otimes \xi - \mathrm{Id}) f \,\mathrm{d}\xi, \quad \mathrm{div}\, u = 0. \qquad (1)$$

Theorem 2: For $De = \phi_N \to 0$, for all $t > 0$, $s_1 > 1/2, s_2 > 0, s_3 > 3/2$

$$\forall \varepsilon > 0 \ \lim_{N\to\infty} \mathbb{P}[\|u_N - u\|_{H^{-s_1}((0,t);H^{-s_1}(\mathbb{R}^3))} + \|f_N - f\|_{H^{-s_2}(0,t;H^{-s_3}(\mathbb{R}^3\times\mathbb{S}^2))} > \varepsilon] = 0,$$

where u solves (1) and f solves

$$\mathrm{div}_\xi((P_{\xi^\perp} hf - \nabla_\xi f) = 0, \qquad \int f \,\mathrm{d}\xi = \int f_0 \,\mathrm{d}\xi.$$

Strategy of the proof

Consider $u_N \approx u_{N,app}$, the solution to

$$-\Delta u_{N,app} + \nabla p_{N,app} = -\frac{\phi_N}{N} \sum_i (S_i + T_i) \nabla \delta_{X_i}, \quad \operatorname{div} u_{N,app} = 0 \qquad \text{in } \mathbb{R}^3,$$

$$T_i := \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i \qquad S_i := \mathcal{R}_{23}^T(\xi_i)\mathcal{R}_2^{-1}(\xi_i)\, T_i := \mathcal{S}(\xi_i) \circ \mathrm{d}B_i$$

Strategy of the proof

Consider $u_N \approx u_{N,app}$, the solution to

$$-\Delta u_{N,app} + \nabla p_{N,app} = -\frac{\phi_N}{N} \sum_i (S_i + T_i) \nabla \delta_{X_i}, \quad \operatorname{div} u_{N,app} = 0 \qquad \text{in } \mathbb{R}^3,$$

$$T_i := \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i \qquad S_i := \mathcal{R}_{23}^T(\xi_i)\mathcal{R}_2^{-1}(\xi_i) T_i := \mathcal{S}(\xi_i) \circ \mathrm{d}B_i$$

Recall

$$\mathrm{d}\xi_i = \xi_i \times \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i + P_{\xi_i^\perp} h \,\mathrm{d}t, \quad \mathrm{d}\mathcal{S}(\xi_i) = \nabla_{\xi_i}\mathcal{S}(\xi_i)\xi_i \times \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i + \cdots \mathrm{d}t$$

Compute by Itô-Stratonovitch conversion

$$\mathbb{E}\left[\int_0^t S_i \,\mathrm{d}t\right] = \mathbb{E}\left[\int_0^t \mathcal{S}(\xi_i) \circ \mathrm{d}B_i\right] = \mathbb{E}\left[\int_0^t \mathcal{S}(\xi_i)\,\mathrm{d}B_i\right] + \frac{1}{2}\mathbb{E}\left[[\mathcal{S}(\xi_i), B_i]_t\right]$$

$$= \cdots = \gamma_{\mathcal{B}} \mathbb{E}\left[\int_0^t (3\xi_i \otimes \xi_i - \mathrm{Id})\,\mathrm{d}s\right]$$

Strategy of the proof

Consider $u_N \approx u_{N,app}$, the solution to

$$-\Delta u_{N,app} + \nabla p_{N,app} = -\frac{\phi_N}{N}\sum_i (S_i + T_i)\nabla\delta_{X_i}, \quad \operatorname{div} u_{N,app} = 0 \qquad \text{in } \mathbb{R}^3,$$

$$T_i := \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i \qquad S_i := \mathcal{R}_{23}^T(\xi_i)\mathcal{R}_2^{-1}(\xi_i)\,T_i := \mathcal{S}(\xi_i) \circ \mathrm{d}B_i$$

Recall

$$\mathrm{d}\xi_i = \xi_i \times \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i + P_{\xi_i^\perp} h\,\mathrm{d}t, \quad \mathrm{d}\mathcal{S}(\xi_i) = \nabla_{\xi_i}\mathcal{S}(\xi_i)\xi_i \times \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i + \cdots \mathrm{d}t$$

Compute by Itô-Stratonovitch conversion

$$\mathbb{E}\left[\int_0^t S_i\,\mathrm{d}t\right] = \mathbb{E}\left[\int_0^t \mathcal{S}(\xi_i) \circ \mathrm{d}B_i\right] = \mathbb{E}\left[\int_0^t \mathcal{S}(\xi_i)\,\mathrm{d}B_i\right] + \frac{1}{2}\mathbb{E}\left[[\mathcal{S}(\xi_i), B_i]_t\right]$$

$$= \cdots = \gamma_{\mathcal{B}}\mathbb{E}\left[\int_0^t (3\xi_i \otimes \xi_i - \mathrm{Id})\,\mathrm{d}s\right]$$

Similarly, $\mathbb{E}[\int_0^t T_i\,\mathrm{d}t] = 0$. Thus, formally, by the Law of Large Numbers,

$$\phi_N^{-1} u_{N,app} \to u, \qquad -\Delta u + \nabla p = \operatorname{div}\sigma := \gamma_{\mathcal{B}} \operatorname{div} \int_{S^2} (3\xi \otimes \xi - \mathrm{Id}) f\,\mathrm{d}\xi$$

Main obstacles

Need to make sense of

$$\begin{aligned} -\Delta u_N + \nabla p_N = 0, \quad \operatorname{div} u_N &= 0 \qquad \text{in } \mathbb{R}^3 \setminus \cup_i \mathcal{B}_i, \\ D u_N(x) &= 0 \qquad \text{in } \cup_i \mathcal{B}_i, \\ \int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu = 0, \qquad \int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu \wedge (x - X_i) &= R^3 \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i, \end{aligned} \tag{2}$$

Main obstacles

Need to make sense of

$$
\begin{aligned}
-\Delta u_N + \nabla p_N = 0, \quad \operatorname{div} u_N = 0 \qquad & \text{in } \mathbb{R}^3 \setminus \cup_i \mathcal{B}_i, \\
D u_N(x) = 0 \qquad & \text{in } \cup_i \mathcal{B}_i, \\
\int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu = 0, \qquad \int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu \wedge (x - X_i) = R^3 \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i, &
\end{aligned}
\tag{2}
$$

Solution: Denote $L_N : (S^2)^N \to \mathcal{L}(\mathbb{R}^{3N}; \dot{H}^1(\mathbb{R}^3))$ the solution operator to (2) with given torques $(T_1, \ldots, T_N)$ instead of $R^3 \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i$.

$$
u_n = U_N' \qquad U_N(t) = \frac{\phi_n}{N} \int_0^t L_N(\xi_1, \ldots, \xi_N) \sqrt{\mathfrak{R}_2(\xi_1, \ldots, \xi_N)} \circ \mathrm{d}(B_1, \ldots B_N)
$$

Main obstacles

Need to make sense of

$$
\begin{aligned}
-\Delta u_N + \nabla p_N = 0, \quad \operatorname{div} u_N &= 0 \qquad \text{in } \mathbb{R}^3 \setminus \cup_i \mathcal{B}_i, \\
D u_N(x) &= 0 \qquad \text{in } \cup_i \mathcal{B}_i, \\
\int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu = 0, \qquad \int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu \wedge (x - X_i) &= R^3 \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i,
\end{aligned}
\tag{2}
$$

Solution: Denote $L_N : (S^2)^N \to \mathcal{L}(\mathbb{R}^{3N}; \dot{H}^1(\mathbb{R}^3))$ the solution operator to (2) with given torques $(T_1, \ldots, T_N)$ instead of $R^3 \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i$.

$$
u_n = U_N' \qquad U_N(t) = \frac{\phi_n}{N} \int_0^t L_N(\xi_1, \ldots, \xi_N) \sqrt{\mathfrak{R}_2(\xi_1, \ldots, \xi_N)} \circ \mathrm{d}(B_1, \ldots B_N)
$$

Need $L_N \in C^1((S^2)^N, \mathcal{L}(\mathbb{R}^{3N}; H))$ for a Hilbert space $H \longrightarrow$ shape derivatives.

Main obstacles

Need to make sense of

$$\begin{aligned} -\Delta u_N + \nabla p_N = 0, \quad \operatorname{div} u_N &= 0 \qquad \text{in } \mathbb{R}^3 \setminus \cup_i \mathcal{B}_i, \\ D u_N(x) &= 0 \qquad \text{in } \cup_i \mathcal{B}_i, \\ \int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu = 0, \qquad \int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu \wedge (x - X_i) &= R^3 \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i, \end{aligned} \tag{2}$$

Solution: Denote $L_N : (S^2)^N \to \mathcal{L}(\mathbb{R}^{3N}; \dot{H}^1(\mathbb{R}^3))$ the solution operator to (2) with given torques $(T_1, \ldots, T_N)$ instead of $R^3 \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i$.

$$u_n = U_N' \qquad U_N(t) = \frac{\phi_n}{N} \int_0^t L_N(\xi_1, \ldots, \xi_N) \sqrt{\mathfrak{R}_2(\xi_1, \ldots, \xi_N)} \circ \mathrm{d}(B_1, \ldots B_N)$$

Need $L_N \in C^1((S^2)^N, \mathcal{L}(\mathbb{R}^{3N}; H))$ for a Hilbert space $H \longrightarrow$ shape derivatives.

Show

$$\|L_N - L_{N,app}\|_{C^1((S^2)^N, \mathcal{L}(\mathbb{R}^{3N}; H_{loc}^{-\frac{1}{2}-}))} = o(1).$$

Key ingredient: Method of reflections that allows to write L_N in terms of single particle solution operators with more explicit shape derivatives.

Main obstacles

Need to make sense of

$$\begin{aligned} -\Delta u_N + \nabla p_N = 0, \quad \operatorname{div} u_N = 0 \qquad & \text{in } \mathbb{R}^3 \setminus \cup_i \mathcal{B}_i, \\ D u_N(x) = 0 \qquad & \text{in } \cup_i \mathcal{B}_i, \end{aligned}$$

$$\int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu = 0, \qquad \int_{\partial \mathcal{B}_i} \sigma[u_N, p_N]\nu \wedge (x - X_i) = R^3 \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i, \tag{2}$$

Solution: Denote $L_N : (S^2)^N \to \mathcal{L}(\mathbb{R}^{3N}; \dot{H}^1(\mathbb{R}^3))$ the solution operator to (2) with given torques $(T_1, \ldots, T_N)$ instead of $R^3 \sqrt{\mathcal{R}_2(\xi_i)} \circ \mathrm{d}B_i$.

$$u_n = U_N' \qquad U_N(t) = \frac{\phi_n}{N} \int_0^t L_N(\xi_1, \ldots, \xi_N) \sqrt{\mathfrak{R}_2(\xi_1, \ldots, \xi_N)} \circ \mathrm{d}(B_1, \ldots B_N)$$

Need $L_N \in C^1((S^2)^N, \mathcal{L}(\mathbb{R}^{3N}; H))$ for a Hilbert space $H \longrightarrow$ shape derivatives.

Show

$$\| L_N - L_{N,app} \|_{C^1((S^2)^N, \mathcal{L}(\mathbb{R}^{3N}; H_{loc}^{-\frac{1}{2}-}))} = o(1).$$

Key ingredient: Method of reflections that allows to write L_N in terms of single particle solution operators with more explicit shape derivatives.

Some open problems

- Overcome restrictions on minimal distance and $\phi_N \to 0$

Some open problems

- Overcome restrictions on minimal distance and $\phi_N \to 0$
- Models of elastic polymers (dumbbell model, Hook and FENE model, ...)

Some open problems

- Overcome restrictions on minimal distance and $\phi_N \to 0$
- Models of elastic polymers (dumbbell model, Hook and FENE model, ...)
- Navier-Stokes equations

Some open problems

- Overcome restrictions on minimal distance and $\phi_N \to 0$
- Models of elastic polymers (dumbbell model, Hook and FENE model, ...)
- Navier-Stokes equations
- Infinitely elongated particles

Some open problems

- Overcome restrictions on minimal distance and $\phi_N \to 0$
- Models of elastic polymers (dumbbell model, Hook and FENE model, ...)
- Navier-Stokes equations
- Infinitely elongated particles
- Full hydrodynamic interactions

Some open problems

- Overcome restrictions on minimal distance and $\phi_N \to 0$
- Models of elastic polymers (dumbbell model, Hook and FENE model, ...)
- Navier-Stokes equations
- Infinitely elongated particles
- Full hydrodynamic interactions
- Translations, including Brownian forces

Some open problems

- Overcome restrictions on minimal distance and $\phi_N \to 0$
- Models of elastic polymers (dumbbell model, Hook and FENE model, ...)
- Navier-Stokes equations
- Infinitely elongated particles
- Full hydrodynamic interactions
- Translations, including Brownian forces

Thank you!